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Efficient Minimum Wages

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Abstract

Should the government raise minimum wages? Further, should the government consider imposing maximum wages? If so, which levels are socially efficient? In a modified version of the Mortensen-Pissarides framework, I find that as productivity increases or as unemployment decreases, an increase in minimum wages could improve social welfare. I also find that the current government proposal of 10.10 dollars per hour is quite close to the socially efficient minimum wage level.

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1 Introduction

As evidenced by the controversy surrounding the U.S. government's proposal to raise the federal minimum wage to 10.10 dollars per hour, the debate among policy makers centers on determining the appropriate level of minimum wages.[1] Several theoretical studies have discussed whether minimum wages can improve social welfare; however, as far as I know, the literature has not addressed which levels are socially efficient, in other words, whether raising the federal minimum wage of 7.25 dollars per hour would improve social welfare.

Even more controversial than minimum wages is the idea of government mandated maximum wages. While theoretical discussions of maximum wages are practically nonexistent, international organizations and several countries have imposed or considered imposing maximum wages for reasons not directly related to economic efficiency. The EU has recently passed regulations to cap bankers' bonuses.[2] Further, as of June 2007, the IMF has imposed wage ceilings in 32 percent of its Poverty Reduction and Growth Facility (PRGF) programs.

Should the government raise minimum wages? Should the government consider imposing maximum wages in addition to minimum wages? If so, which levels are socially efficient? I attempt to answer these questions in a modified version of the Mortensen-Pissarides framework. I find that minimum wages can improve social welfare, provided that a worker's bargaining power is less than the matching elasticity.[3] Further, as labor productivity increases or as unemployment decreases, an increase in minimum wages improves social welfare. I also find that setting maximum wage levels can also improve social welfare, provided that a worker's bargaining power is greater

[1] See http://www.dol.gov/whd/flsa/nprm-eo13658/.

[2] The cap has been effective since January 1, 2014. See Financial Times, March 20, 2013.

[3] See Acemoglu (2001).

than the matching elasticity. Finally, assuming that the productivity growth rate and the rate of inflation follow the average rates for 1990-2013, I find that the socially efficient minimum wage level for 2014-2017 is remarkably close to the level proposed by the government.

Typically, derivation of the optimal wages in the models of minimum wages is complicated due to non-differentiabilities associated with wage floors (see the literature cited below). To get around such difficulties, I rely on the concept of effective bargaining power – the level of bargaining power that equates laissez-faire wages with government imposed wages. A worker's effective bargaining power increases with binding minimum wages and decreases with binding maximum wages. To be socially efficient, the effective bargaining power must equal the matching elasticity, which is the modified Hosios condition with binding wages. When the actual level of bargaining power is less than the matching elasticity, the government should impose binding minimum wages to raise the effective bargaining power. When the actual level of bargaining power is greater than the matching elasticity, the government should impose binding maximum wages to reduce the effective bargaining power. I derive a closed form expression for optimal minimum or maximum wages and show that optimal wage levels depend on productivity, income from nonmarket activities, the matching elasticity, and the unemployment rate.

This paper is related to recent studies that discuss the welfare or employment effects of minimum wages in the labor market with varying degrees of complexity: monopsonistic competition (Manning (2003)), efficiency wages (Rebitzer and Taylor (1995)), a wage bargaining model with skilled and unskilled labor (Cahuc, Saint-Martin, and Zylberberg (2001)), and a matching model with low-wage and high-wage jobs (Acemoglu (2001)). The model in this paper is simpler than the matching models cited above, but still finds that minimum wages can improve social welfare. Further, the model's implications on the effects of changes in the parameters, for example, productivity and the job separation rate, on the optimal minimum wage levels are more straightforward.

This paper also shows that maximum wages are available to policy makers as an instrument that can be used to improve social welfare. Imposing maximum wages would be particularly relevant in situations where alternative means of transferring resources are not as effective for various socioeconomic or institutional reasons.

2 The Basic Model

The model builds on the standard Mortensen-Pissarides framework and is extended to incorporate binding minimum and maximum wages.[4] There are u unemployed workers looking for jobs and v vacancies posted by firms looking for workers. An unemployed worker is matched with a suitable job at rate α_w per unit time, and a firm with a vacancy is matched with a suitable worker at rate α_e per unit time. Following Pissarides (2000), I assume a matching function $m(u, v)$ such that

$$\alpha_w = m(u, v)/u \text{ and } \alpha_e = m(u, v)/v. \quad (1)$$

The function $m(\bullet, \bullet)$ is increasing in both arguments, concave, and homogeneous of degree 1. To be specific, I assume the matching function to have the following Cobb-Douglas form:

$$m(u, v) = m_0 u^{\delta} v^{1-\delta}, \quad (2)$$

where $m_0 > 0$ and $\delta \in (0, 1)$ is the elasticity of the matching function with respect to unemployment.

When an unemployed worker is matched to a firm with a vacancy, the

[4] See Pissarides (2000) and Rogerson, Shimer, and Wright (2005) for various applications of the framework.

worker-firm pair produces y units of output and splits the surplus through Nash bargaining. An employed worker receives w, where $w < y$. When the pair breaks up, the firm either creates a vacancy and looks for suitable workers or leaves the market. A pair breaks up at rate μ per unit time. A firm with a vacancy produces no output and incurs a cost of holding a vacancy k. An unemployed worker's income from non-market activities amounts to zy, where $z \in (0, 1)$. The number of workers is normalized to one. All stochastic events are independent. The value of unemployment U and that of employment W are:

$$\rho U = zy + \alpha_w (W - U); \tag{3}$$

$$\rho W = w - \mu (W - U), \tag{4}$$

where $\rho > 0$ is the discount rate. The value of a vacancy V and that of a match J are:

$$\rho V = -k + \alpha_e (J - V); \tag{5}$$

$$\rho J = y - w - \mu (J - V). \tag{6}$$

As free entry decreases the value of a vacancy to zero, $V = 0$ in equilibrium. According to (5) and (6),

$$0 = \alpha_e (y - w) - k(\rho + \mu). \tag{7}$$

2.1 Laissez-faire Wages

A worker-firm pair maximizes $[W - U]^{\beta} [J - V]^{1-\beta}$ for any given ρ, μ, α_w, α_e, z, y, and β, where $\beta \in (0, 1)$ is the worker's bargaining power. Thus, the laissez-faire wage w satisfies

$$0 = -(1 - \beta)(w - zy)(\rho + \mu) + \beta (y - w)(\rho + \mu + \alpha_w). \tag{8}$$

2.2 Binding Wages

I now consider a case where the government imposes a binding wage $\hat{w}$, where $\hat{w} < y$. Typically, a maximization problem with binding minimum (or maximum) wages involves non-differentiable regions. To avoid difficulties associated with non-differentiability, I use the notion of effective bargaining power.

Definition: $\hat{\beta}$ is the effective bargaining power if $\hat{\beta}$ satisfies (8) for any given ρ, μ, α_w, z, y, and $\hat{w}$.

The following proposition describes the relationship between the effective bargaining power $\hat{\beta}$ and the actual bargaining power β.

Proposition 1: Let w be the laissez-faire wage and $\hat{w}$ be the binding wage such that $\hat{w} = w(1+\phi)$, where $|\phi| < 1$. Then, $\hat{\beta}$ increases with $\hat{w}$. Furthermore, $\hat{\beta}$ can be expressed as follows:

$$\frac{1}{\hat{\beta}} = \left[\left(\Gamma\left[1+\left(\frac{1}{\beta}-1\right)\Gamma\right]^{-1} + \frac{z\Gamma}{1-z}\right)(1+\phi) - \frac{z\Gamma}{1-z}\right]^{-1} - \Gamma^{-1} + 1, \quad (9)$$

where $\Gamma = (\rho+\mu)\,/\,(\rho+\mu+\alpha_w)$.

Proof: According to (8), I have:

$$w = \left[1+\left(\frac{1}{\beta}-1\right)\Gamma\right]^{-1}\left[1+\left(\frac{1}{\beta}-1\right)z\Gamma\right]y, \quad (10)$$

where Γ is defined in (9). From (10), let us define f such that $f(\beta) = [1+(1/\beta-1)\,\Gamma]^{-1}[1+(1/\beta-1)\,z\Gamma]$. Note that $f : (0,1) \to (0,1)$. For convenience, I rewrite f as $f(\beta) = [1+(1/\beta-1)\,\Gamma]^{-1}(1-z)+z$. Since f is one-to-one and onto, f^{-1} exists and for any given $\hat{w}$ and y, there is a unique $\hat{\beta}$, which satisfies (8).[5] With $\hat{\beta}$, $\hat{w}$ satisfies (8) given ρ, μ, α_w, z, and y. By

[5] f is one-to-one since if $[1+(1/\beta_1-1)\,\Gamma]^{-1}(1-z)+z = [1+(1/\beta_2-1)\,\Gamma]^{-1}(1-z)+z$,

totally differentiating (8) with $(\hat{\beta},\, \hat{w})$ instead of (β, w), I have:

$$\frac{d\hat{\beta}}{d\hat{w}} = \frac{\rho + \mu + \hat{\beta}\alpha_w}{y(1-z)(\rho+\mu) + (y-\hat{w})\alpha_w} > 0. \tag{11}$$

Furthermore, from (8) with $(\hat{\beta},\, \hat{w})$, I have:

$$\hat{w} = \left[1 + \left(\frac{1}{\hat{\beta}} - 1\right)\Gamma\right]^{-1}\left[1 + \left(\frac{1}{\hat{\beta}} - 1\right)z\Gamma\right]y, \tag{12}$$

where Γ is defined in (9). Dividing (12) by (10), then rearranging terms using $\hat{w} = w(1+\phi)$, I get (9). Q.E.D.

Proposition 1 works for cases with wage floors as well as wage ceilings. If $\phi > 0$, then $\hat{w} > w$ and $\hat{w}$ becomes a binding minimum wage. If $\phi < 0$, then $\hat{w} < w$ and $\hat{w}$ becomes a binding maximum wage. When minimum or maximum wages are not binding, the effective bargaining power equals the actual level of bargaining power. That is, if $\hat{w}$ equals w, ϕ is zero. Then, $\hat{\beta} = \beta$ according to (9).

2.3 Steady State Equilibrium

Let u denote the steady state rate of unemployment. In the steady state, $(1-u)\mu$ workers lose jobs and $\alpha_w u$ workers find jobs at each moment. Equating these two numbers, I have:

$$u = \mu / \left(\mu + \alpha_w\right). \tag{13}$$

The steady state equilibrium with $(\hat{\beta},\, \hat{w})$ is characterized by (1), (2), (7) with $\hat{w}$ instead of w, (8) with $(\hat{\beta},\, \hat{w})$ instead of (β, w), (9), and (13).

$\beta_1 = \beta_2$. Note that if $f(\beta) = [1+(1/\beta-1)\,\Gamma]^{-1}\,(1-z)+z = x$, $\beta = [1-z+(1-x)\alpha_w/(\rho+\mu)]^{-1}(x-z)$. f is onto since $f([1-z+(1-x)\alpha_w/(\rho+\mu)]^{-1}(x-z)) = x$.

3 Efficiency

3.1 Social Welfare

In the steady state with binding wages, $(1-u)$ employed workers receive $(1-u)\hat{w}$, and u unemployed workers receive uzy. With the linear utility function, welfare of all the workers becomes $(1-u)\,\hat{w}+uzy$. As $(1-u)$ firms are matched with workers and v firms incur vacancy holding costs vk, total profits of all the firms in the economy equal $(1-u)\,(y-\hat{w})-vk$. Following Hosios (1990), I let $\rho \to 0$. Then, total profits become zero.[6] Thus, social welfare (SW) becomes:[7]

$$SW = \{1-u(1-z)\}y - vk. \tag{14}$$

According to (14), social welfare decreases with unemployment and vacancies for any given k, z,and y.

3.2 The Social Planner's Problem

Given m_0, δ, μ, k, z, and y, the social planner chooses $\{u, v\}$ to maximize SW subject to constraints (1), (2), and (13). The FOC of the social planner's problem becomes:

$$0 = \left(\frac{1}{\delta} - 1\right)(1-z)y - k\left(\frac{1}{\delta}\mu + \alpha_e \frac{v}{u}\right)/\alpha_e. \tag{15}$$

According to (1), (2), (13), and (15), for any given m_0, δ, μ, k, z, and y, the social welfare maximizing values of α_w, α_e, u, and v are uniquely determined.

[6] According to (1), (2), and (7) with $\hat{w}$, total profits become $\rho vk/\mu$.

[7] $(1-u)\,\hat{w} = (1-u)y - vk$ since $(1-u)\,(y-\hat{w}) - vk = 0$; $(1-u)y - vk + uzy$ $= \{1-u(1-z)\}y - vk$.

From (7) with $\hat{w}$, (8) with $(\hat{\beta}, \hat{w})$, and (13), I have: as $\rho \to 0$,

$$0 = \left(\frac{1}{\hat{\beta}} - 1\right)(1-z)y - k\left(\frac{1}{\hat{\beta}}\mu + \alpha_e \frac{v}{u}\right)/\alpha_e. \tag{16}$$

From (15) and (16), I can prove the following proposition by matching coefficients.

Proposition 2: For the steady state equilibrium with binding wages to be socially efficient, I need:

$$\delta = \hat{\beta}. \tag{17}$$

(17) is the modified Hosios condition with binding wages. The following proposition makes it operational.

Proposition 3: Let w be the laissez-faire wage which satisfies (8) with $\rho \to 0$. Let $\hat{w}^*$ be the binding optimal wage. Then, I have:

$$\hat{w}^* = \left[1 + \left(\frac{1}{\delta} - 1\right)u\right]^{-1}\left[1 + \left(\frac{1}{\delta} - 1\right)uz\right]y. \tag{18}$$

The binding optimal wage can be expressed in terms of the laissez-faire wage: $\hat{w}^* = w(1+\phi^*)$, where

$$\phi^* = \left(\frac{1}{\beta} - \frac{1}{\delta}\right)u(1-z)/\left[\left\{1 + \left(\frac{1}{\delta} - 1\right)u\right\}\left\{1 + \left(\frac{1}{\beta} - 1\right)uz\right\}\right]. \tag{19}$$

Proof: Substituting (13) and (17) into (12) with $\rho \to 0$, I get (18). From (18) and (10) with $\rho \to 0$, I get (19). Q.E.D.

(18) is the closed form expression for the binding optimal wage. The binding optimal wage depends on productivity, income from nonmarket activities, the matching elasticity, and the unemployment rate. More specifically, the

binding optimal wage increases with productivity, income from nonmarket activities, and the matching elasticity; it decreases with the unemployment rate.

To understand the relationship between binding optimal wages and laissez-faire wages, and the direction of the optimal policy, I consider the following three cases. First, suppose $\delta = \beta$. According to (10) with $\rho \to 0$ and (18), or (19), I have: $\hat{w}^* = w$ and $\phi^* = 0$. This is the Hosios (1990) finding that with $\delta = \beta$, the laissez-faire equilibrium is socially efficient. Second, suppose $\beta < \delta$. According to (19), I have: $\hat{w}^* > w$ and $\phi^* > 0$. In this case, the laissez-faire wage is lower than the optimal minimum wage $\hat{w}^*$. Raising the binding minimum wage increases the effective bargaining power $\hat{\beta}$ toward δ. Finally, suppose $\delta < \beta$. According to (19), I have: $\hat{w}^* < w$ and $\phi^* < 0$. In this case, the laissez-faire wage is higher than the optimal maximum wage $\hat{w}^*$. Lowering the binding maximum wage decreases the effective bargaining power $\hat{\beta}$ toward δ.

4 Efficient Minimum Wages in the U.S.

According to Proposition 3, the optimal minimum wage increases as productivity increases or as unemployment decreases. Thus, when formulating a minimum wage policy, the government should incorporate such information on productivity and unemployment. I first discuss whether the U.S. minimum wage policy has been consistent with the policy recommendations in this paper. The U.S. government changed the federal minimum wages seven times during the twenty four-year period starting 1990: April 1, 1990, April 1, 1991, Oct. 1, 1996, Sept. 1, 1997, July 24, 2007, July 24, 2008, and July 24, 2009.[8] Since the government set minimum wages would prevail for 3.5 years on average, I divide the 1990-2013 period into six 4-year periods. Note

[8] The nonminal minimum wages corresponding to these dates are \$3.80, \$4.25, \$4.75, \$5.15, \$5.85, \$6.55 and \$7.25, respectively. See the U.S. Dept. of Labor website <http://www.dol.gov/whd/minwage/chart.htm>.

that minimum wages did not change for a ten year period between 1997 and 2007, and thus the 1998-2001 and 2002-2005 periods do not involve any minimum wage changes. I presume the difference between the optimal minimum wage and the actual one to be the smallest in the 1998-2001 period as this period is further away from the next minimum wage change than any other four-year period since 1990.

According to the BLS website, the average U.S. per capita output figures are: 82.30 for 1998-2001, 92.36 for 2002-2005, 98.42 for 2006-2009, and 105.81 for 2010-2013; the average unemployment rates for the corresponding periods are: 4.4, 5.6, 6.1, and 8.5 percent; the average real minimum wages (in 2013 dollars) for the corresponding periods are: 7.09, 6.42, 6.47, and 7.46 dollars.[9] I assume that the average minimum wage coincides with the optimal minimum wage for the 1998-2001 period. To account for productive inputs besides labor, I assume that the optimal minimum wage is proportional to the one described in (18). Following Hall and Milgrom (2008) and Pissarides (2009), I assume that $\delta = 0.5$ and $z = 0.71$. With such assumptions, I find that the optimal minimum wages for the 2002-2005, 2006-2009 and 2010-2013 periods are 7.93, 8.43, and 9.02 dollars.[10] Thus, the decrease in the average real minimum wage to 6.42 dollars for 2005-2009 does not seem consistent with the optimal policy in this paper. However, the subsequent increases seem consistent, even though they are below the optimal level.

Finally, I calculate the optimal minimum wages for the 2014-2017 period. I assume that for 2014-2017, productivity growth and unemployment follow

[9]Following Shimer (2005), I measure productivity using real average output per person in the nonfarm business sector, constructed by the BLS Major Sector Productivity and Costs program. I convert the nominal federal minimum wages into the real minimum wages in 2013 dollars using the CPI. For productivity, unemployment, and CPI data, see the BLS website <http://data.bls.gov/pdq/SurveyOutputServlet>.

[10]Substituting the average U.S. per capita output and unemployment data for various periods with $\delta = 0.5$ and $z = 0.71$ into (18), I have: 90.94 for 2002-2005, 96.78 for 2006-2009, 103.41 for 2010-2013. Since the average real minimum wage for 1998-2001 is 7.09 dollars per hour, I have: $7.93 = 7.09 \times 90.94/81.31$, $8.43 = 7.09 \times 96.78/81.31$ and $9.02 = 7.09 \times 103.41/81.031$.

the average rates for 1990-2013: 2.03 per cent and 6.13 per cent, respectively. I find the optimal real minimum wage rate for 2014-2017 to be 9.67 dollars.[11] Since the minimum wage rate is set nominally, I convert the real minimum wage rate into nominal terms by considering expected future price changes. I assume that inflation for 2014-2017 follows the average rate for 1990-2013, which is 2.55 per cent. I consider two cases. First, suppose that the government sets the minimum wage rate nominally in 2014 and adjusts it annually to inflation until 2017. In this case, the optimal minimum wage rate is 9.92 dollars. Second, suppose that the government sets the minimum wage rate nominally in 2014 and fixes it for the entire 2014-2017 period. In this case, the optimal nominal minimum wage rate becomes 10.25 dollars.[12] The former is slightly lower and the latter is slightly higher than 10.10 dollars. But they are remarkably close to the current government's proposal.

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[11] According to the BLS website, the output per capita in 2013 is 107.20. Since the output is expected to grow at 2.03 percent, the average output is expected to be 112.76 for 2014-2017. Thus, the optimal real minimum wage becomes $9.67 in 2013 prices.

[12] $10.25 = 9.67 \times (\sum_{i=1}^{4} 1.0255^{i}/4)$.

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